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Dynamic General Equilibrium Modelling

Computational Methods and Applications

With 63 Figures and 24 Tables

Springer

UNIVERSITÄT ST. GALLEN
HOCHSCHULE FÜR WIRTSCHAFTS-,
RECHTS- UND SOZIALWISSENSCHAFTEN
BIBLIOTHEK

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