Mathematica By Example

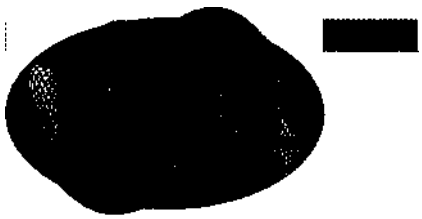
Third Edition

Martha L. Abell and James P. Braselton

ELSEVIER
ACADEMIC
PRESS

Amsterdam Boston Heidelberg London New York Oxford Paris
San Diego San Francisco Singapore Sydney Tokyo

Contents



Preface ix

1 Getting Started 1

1.1 Introduction to Mathematica . . . 1
A Note Regarding Different Versions of Mathematica . . . 3
1.1.1 Getting Started with Mathematica . . . 3
Preview . . . 9
1.2 Loading Packages . . . 10
A Word of Caution . . . 13
1.3 Getting Help from Mathematica . . . 14
Mathematica Help . . . 18
The Mathematica Menu . . . 22

2 Basic Operations on Numbers, Expressions, and Functions 23

2.1 Numerical Calculations and Built-in Functions . . . 23
2.1.1 Numerical Calculations . . . 23
2.1.2 Built-in Constants . . . 26
2.1.3 Built-in Functions . . . 27
A Word of Caution . . . 30
2.2 Expressions and Functions: Elementary Algebra . . . 31
2.2.1 Basic Algebraic Operations on Expressions . . . 31
2.2.2 Naming and Evaluating Expressions . . . 37
Two Words of Caution . . . 38
2.2.3 Defining and Evaluating Functions . . . 39
2.3 Graphing Functions, Expressions, and Equations . . . 45

2.3.1 Functions of a Single Variable . . . 45
2.3.2 Parametric and Polar Plots in Two Dimensions . . . 58
2.3.3 Three-Dimensional and Contour Plots; Graphing Equations 65
2.3.4 Parametric Curves and Surfaces in Space . . . 75
2.4 Solving Equations . . . 81
2.4.1 Exact Solutions of Equations . . . 81
2.4.2 Approximate Solutions of Equations . . . 90

3 Calculus 97
3.1 Limits . . . 97
3.1.1 Using Graphs and Tables to Predict Limits . . . 98
3.1.2 Computing Limits . . . 99
3.1.3 One-Sided Limits . . . 102
3.2 Differential Calculus . . . 104
3.2.1 Definition of the Derivative . . . 104
3.2.2 Calculating Derivatives . . . 107
3.2.3 Implicit Differentiation . . . 110
3.2.4 Tangent Lines . . . 111
3.2.5 The First Derivative Test and Second Derivative Test . . . 123
3.2.6 Applied Max/Min Problems . . . 128
3.2.7 Antidifferentiation . . . 138
3.3 Integral Calculus . . . 141
3.3.1 Area . . . 141
3.3.2 The Definite Integral . . . 147
3.3.3 Approximating Definite Integrals . . . 153
3.3.4 Area . . . 156
3.3.5 Arc Length . . . 162
3.3.6 Solids of Revolution . . . 167
3.4 Series . . . 173
3.4.1 Introduction to Sequences and Series . . . 173
3.4.2 Convergence Tests . . . 178
3.4.3 Alternating Series . . . 182
3.4.4 Power Series . . . 184
3.4.5 Taylor and Maclaurin Series . . . 187
3.4.6 Taylor's Theorem . . . 192
3.4.7 Other Series . . . 196
3.5 Multi-Variable Calculus . . . 198
3.5.1 Limits of Functions of Two Variables . . . 198
3.5.2 Partial and Directional Derivatives . . . 201
3.5.3 Iterated Integrals . . . 218

4 Introduction to Lists and Tables 229
4.1 Lists and List Operations . . . 229
4.1.1 Defining Lists . . . 229
4.1.2 Plotting Lists of Points . . . 233
4.2 Manipulating Lists: More on Part and Map . . . 248
4.2.1 More on Graphing Lists; Graphing Lists of Points Using Graphics Primitives . . . 258
4.2.2 Miscellaneous List Operations . . . 267
4.3 Mathematics of Finance . . . 267
4.3.1 Compound Interest . . . 268
4.3.2 Future Value . . . 270
4.3.3 Annuity Due . . . 271
4.3.4 Present Value . . . 273
4.3.5 Deferred Annuities . . . 274
4.3.6 Amortization . . . 275
4.3.7 More on Financial Planning . . . 280
4.4 Other Applications . . . 287
4.4.1 Approximating Lists with Functions . . . 287
4.4.2 Introduction to Fourier Series . . . 294
4.4.3 The Mandelbrot Set and Julia Sets . . . 308

5 Matrices and Vectors 327
5.1 Nested Lists: Introduction to Matrices, Vectors, and Matrix Operations . . . 327
5.1.1 Defining Nested Lists, Matrices, and Vectors . . . 327
5.1.2 Extracting Elements of Matrices . . . 334
5.1.3 Basic Computations with Matrices . . . 337
5.1.4 Basic Computations with Vectors . . . 342
5.2 Linear Systems of Equations . . . 349
5.2.1 Calculating Solutions of Linear Systems of Equations . . . 349
5.2.2 Gauss-Jordan Elimination . . . 355
5.3 Selected Topics from Linear Algebra . . . 362
5.3.1 Fundamental Subspaces Associated with Matrices . . . 362
5.3.2 The Gram-Schmidt Process . . . 364
5.3.3 Linear Transformations . . . 370
5.3.4 Eigenvalues and Eigenvectors . . . 373
5.3.5 Jordan Canonical Form . . . 377
5.3.6 The QR Method . . . 381
5.4 Maxima and Minima Using Linear Programming . . . 384

5.4.1 The Standard Form of a Linear Programming Problem . . . 384
5.4.2 The Dual Problem . . . 386
5.5 Selected Topics from Vector Calculus . . . 393
5.5.1 Vector-Valued Functions . . . 393
5.5.2 Line Integrals . . . 402
5.5.3 Surface Integrals . . . 407
5.5.4 A Note on Nonorientability . . . 411

Differential Equations 429
6.1 First-Order Differential Equations . . . 429
6.1.1 Separable Equations . . . 429
6.1.2 Linear Equations . . . 434
6.1.3 Nonlinear Equations . . . 444
6.1.4 Numerical Methods . . . 448
6.2 Second-Order Linear Equations . . . 454
6.2.1 Basic Theory . . . 454
6.2.2 Constant Coefficients . . . 455
6.2.3 Undetermined Coefficients . . . 462
6.2.4 Variation of Parameters . . . 467
6.3 Higher-Order Linear Equations . . . 470
6.3.1 Basic Theory . . . 470
6.3.2 Constant Coefficients . . . 470
6.3.3 Undetermined Coefficients . . . 473
6.3.4 Laplace Transform Methods . . . 485
6.3.5 Nonlinear Higher-Order Equations . . . 498
6.4 Systems of Equations . . . 498
6.4.1 Linear Systems . . . 498
6.4.2 Nonhomogeneous Linear Systems . . . 515
6.4.3 Nonlinear Systems . . . 519
6.5 Some Partial Differential Equations . . . 538
6.5.1 The One-Dimensional Wave Equation . . . 538
6.5.2 The Two-Dimensional Wave Equation . . . 544
6.5.3 Other Partial Differential Equations . . . 556