

THE CONCEPT OF NUMBER

*A Chapter in the History of Mathematics,
with Applications of Interest to Teachers*

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<u>Chapter</u>	<u>Page</u>
I. THE HISTORICAL DEVELOPMENT OF NUMERATION	1
1.1 Numbers as Adjectives	2
1.2 Correspondence with other Sets: Tallying and Finger Numbers	6
1.3 The Modes of Forming a Written Number Sequence	12
1.4 Number Words - The Spoken Number Sequence	22
1.5 The Number Symbols	35
1.6 Analysis of Examples	50
II. ELEMENTARY OPERATIONS WITH NUMBERS	59
11.1 The First Mechanical Computer: The Abacus	60
11.2 The Arithmetic of the Middle Ages and the Renaissance	77
11.3 Other Interesting Multiplication and Division Methods	87
11.4 The Binary Number System	95
III. HIGHER ARITHMETIC	102
111.1 The Logical Construction of a Number System	102
111.2 The Historical Development of Artificial Numbers	112
111.3 Basic Operations with Artificial Numbers	121
IV. ALGEBRA AND NUMBER THEORY	136
IV.1 Linear and Quadratic Equations	137
IV.2 Cubic and Quartic Equations	149
IV.3 Numbers and their Divisors	158
IV.4 Diophantine Arithmetic	172
V. CONTRIBUTIONS TO THE NUMBER CONCEPT SINCE THE BEGINNING OF THE 19th CENTURY	183
V.1 Equations and their Roots	184
V.2 Algebra and the Number Concept	190
V.3 Infinity and the Number Concept	195
Appendix A : Finger Multiplication	208
Appendix B : Method of Extracting Cube Roots	209
Bibliography	211
Answers to Selected Problems	213
Index	214

LIST OF FIGURES

<u>Figure</u>	<u>Page</u>
1. The Number Scale as a Staircase	26
2. Addition and Subtraction on the Greek Abacus	62
3. Blank Copies of Greek and Roman Abaci	64
4. Multiplication on the Roman Abacus	66
5. The Arc Abacus Showing Gerbert's Apices	70
6. Multiplication on the Arc Abacus	70
7. Golden and Iron Division	72
8. Blank Copy of Arc Abacus	73
9. Later Form of Loose-Counter Abacus	74
10. The Chinese Abacus	76
11. An Early-Addition Method	77
12. A Problem in Subtraction	78
13. Two Forms of Chessboard Multiplication	80
14. Castle Method of Multiplication	81
15. Cross Multiplication	81
16. Multiplication of Two Digit Numbers	82
17. The Method of the Quadrilateral	82
18. The Gelosia or Net Method of Multiplication	83
19. The Teen Rule	84
20. Some Examples of a Rapid Calculation Method	84
21. The Galley or Scratch Method of Division	85
22. An Ancient Format for Long Division	86
23. An Early Printed Form of Long Division	87
24. The Contracted Plan of Division	87
25. Examples of Egyptian Multiplication	89
26. A Modified Form of Egyptian Multiplication	90
27. Egyptian Division	90
28. A Babylonia [^] Multiplication Table	91
29. Russian Multiplication	92
30. Comparison of Decimal and Binary Numbers	97
31. Binary Addition and Multiplication Tables	97
32. Some Examples of Binary Computations	98
33. The Number System and The Gaussian Plane	111
34. A 16th Century Table of Square Roots	114
35. The Scratch Method of Extracting Square Hoots	126
36. Long Division Methods of Extracting Square Roots	127
37. Pascal's Triangle	127
38. Burgi's Logarithms	129
39. Number Classifications	188