

Freddy Delbaen • Walter Schachermayer

The Mathematics of Arbitrage

UNIVERSITÄT
LIECHTENSTEIN
Bibliothek

4y Springer

Contents

Part I A Guided Tour to Arbitrage Theory

1	The Story in a Nutshell	3
1.1	Arbitrage	3
1.2	An Easy Model of a Financial -Market	4
1.3	Pricing by No-Arbitrage	5
1.4	Variations of the Example	7
*	1.5 Martingale Measures	7
1.6	The Fundamental Theorem of Asset Pricing	8
2	Models of Financial Markets on Finite Probability Spaces	11
2.1	Description of the Model	11
2.2	No-Arbitrage and the Fundamental Theorem of Asset Pricing	16
2.3	Equivalence of Single-period with Multiperiod Arbitrage	22
2.4	Pricing by No-Arbitrage	23
2.5	Change of Numeraire	27
2.6	Kramkov's Optional Decomposition Theorem	31
3	Utility Maximisation on Finite Probability Spaces	33
3.1	The Complete Case	34
3.2	The Incomplete Case	41
3.3	The Binomial and the Trinomial Model	45
4	Bachelier and Black-Scholes	57
4.1	Introduction to Continuous Time Models	57
4.2	Models in Continuous Time	57
4.3	Bachelier's Model	58
4.4	The Black-Scholes Model	60

5	The Kreps-Yan Theorem	71
5.1	A General Framework	71
5.2	No Free Lunch!	76
6	The Dalang-Morton-Willinger Theorem	85
6.1	Statement of the Theorem	85
6.2	The Predictable Range	86
6.3	The Selection Principle	89
6.4	The Closedness of the Cone C	92
6.5	Proof of the Dalang-Morton-Willinger Theorem for $T = 1$	94
6.6	A Utility-based Proof of the DMW Theorem for $T = 1$	96
6.7	Proof of the Dalang-Morton-Willinger Theorem for $T > 1$ by Induction on T	102
6.8	Proof of the Closedness of K in the Case $T > 1$	103
6.9	Proof of the Closedness of C in the Case $T > 1$ under the (NA) Condition	105
6.10	Proof of the Dalang-Morton-Willinger Theorem for $T > 1$ using the Closedness of C	107
6.11	Interpretation of the L^∞ -Bound in the DMW Theorem	108
7	A Primer in Stochastic Integration	111
7.1	The Set-up	111
7.2	Introductory on Stochastic Processes	112
7.3	Strategies, Semi-martingales and Stochastic Integration	117
8	Arbitrage Theory in Continuous Time: an Overview	129
8.1	Notation and Preliminaries	129
8.2	The Crucial Lemma	131
8.3	Sigma-martingales and the Non-locally Bounded Case	140

Part II The Original Papers

9	A General Version of the Fundamental Theorem of Asset Pricing (1994)"	149
9.1	Introduction	149
9.2	Definitions and Preliminary Results	155
9.3	No Free Lunch with Vanishing Risk	160
9.4	Proof of the Main Theorem	164
9.5	The Set of Representing Measures	181
9.6	No Free Lunch with Bounded Risk	186
9.7	Simple Integrands	190
9.8	Appendix: Some Measure Theoretical Lemmas	202

10	A Simple Counter-Example to Several Problems in the Theory of Asset Pricing (1998)	207
	10.1 Introduction and Known Results.....	207
	10.2 Construction of the Example.....	210
	10.3 Incomplete Markets.....	212
11	The No-Arbitrage Property under a Change of Numeraire (1995)	217
	11.1 Introduction.....	217
	11.2 Basic Theorems.....	219
	11.3 Duality Relation.....	222
	11.4 Hedging and Change of Numeraire.....	225
12	The Existence of Absolutely Continuous Local Martingale Measures (1995)	231
	12.1 Introduction.....	231
	12.2 The Predictable Radon-Nikodym Derivative.....	235
	12.3 The No-Arbitrage Property and Immediate Arbitrage.....	239
	12.4 The Existence of an Absolutely Continuous Local Martingale Measure	244
13	The Banach Space of Workable Contingent Claims in Arbitrage Theory (1997)	251
	13.1 Introduction.....	251
	13.2 Maximal Admissible Contingent Claims.....	255
	13.3 The Banach Space Generated by Maximal Contingent Claims.....	261
	13.4 Some Results on the Topology of Q	266
	13.5 The Value of Maximal Admissible Contingent Claims on the Set M^e	272
	13.6 The Space Q under a Numeraire Change.....	274
	13.7 The Closure of $Q^{\circ\circ}$ and Related Problems.....	276
14	The Fundamental Theorem of Asset Pricing for Unbounded Stochastic Processes (1998)	279
	14.1 Introduction.....	279
	14.2 Sigma-martingales.....	280
	14.3 One-period Processes.....	284
	14.4 The General Revalued Case.....	294
	14.5 Duality Results and Maximal Elements.....	305
15	A Compactness Principle for Bounded Sequences of Martingales with Applications (1999)	319
	15.1 Introduction.....	319
	15.2 Notations and Preliminaries.....	326

XVI Contents

15.3 An Example. 332

15.4 A Substitute of Compactness
for Bounded Subsets of H^1 334

15.4.1 Proof of Theorem 15.A. 335

15.4.2 Proof of Theorem 15.C. 337

15.4.3 Proof of Theorem 15.B. 339

15.4.4 A proof of M. Yor's Theorem. 345

15.4.5 Proof of Theorem 15.D. 346

15.5 Application. 352

Part III Bibliography

References. 359