

Eric Jondeau, Ser-Huang Poon
and Michael Rockinger

Financial Modeling Under Non-Gaussian Distributions

Springer

Contents

Part I Financial Markets and Financial Time Series

1	Introduction	3
1.1	Financial markets and financial time series	3
1.2	Econometric modeling of asset returns	4
1.3	Applications of non-Gaussian econometrics	5
1.4	Option pricing with non-Gaussian distributions	5
2	Statistical Properties of Financial Market Data	7
2.1	Definitions of returns	7
2.1.1	Simple returns	8
2.1.2	Log-returns	8
2.1.3	Stylized facts	9
2.2	Distribution of returns	10
2.2.1	Moments of a random variable	10
2.2.2	Empirical moments	14
2.2.3	Testing for normality	16
2.3	Time dependency	21
2.3.1	Serial correlation in returns	22
2.3.2	Serial correlation in volatility	23
2.3.3	Volatility asymmetry	25
2.3.4	Time-varying higher moments	26
2.4	Linear dependence across returns	26
2.4.1	Pearson's correlation coefficient	27
2.4.2	Test for equality of two correlation coefficients	28
2.4.3	Test for equality of two correlation matrices	30
2.5	Multivariate higher moments	31
2.5.1	Multivariate co-skewness and co-kurtosis	31
2.5.2	Computing moments of portfolio returns	32

3 Functioning of Financial Markets and Theoretical Models for Returns 33

3.1 Functioning of financial markets. 34

3.1.1 Organization of financial markets. 34

3.1.2 Examples of orders. 37

3.1.3 Components of the bid-ask spread. 39

3.2 Mandelbrot and the stable distribution. 39

3.2.1 A puzzling result. 40

3.2.2 Stable distribution. 41

3.3 Clark's subordination model. 44

3.3.1 The idea of the model. 44

3.3.2 The density of returns under subordination. 46

3.4 A bivariate mixture-of-distribution model for return and volume. 48

3.4.1 A microstructure model for information arrivals. 48

3.4.2 Implications of the mixture of distributions hypothesis. 53

3.4.3 Testing the mixture of distribution hypothesis. 57

3.4.4 Extensions. 61

3.5 A model of prices and quotes in a quote-driven market. 62

3.5.1 A model based on the trade flow. 63

3.5.2 Estimating the parameters. 66

3.5.3 The quote process. 68

3.5.4 Extension to the liquidation of a large portfolio. 73

Part II Econometric Modeling of Asset Returns

4 Modeling Volatility 79

4.1 Volatility at lower frequencies. 79

4.2 ARCH model. 81

4.2.1 Forecasting. 81

4.2.2 Kurtosis of an ARCH model. 82

4.2.3 Testing for ARCH effects. 82

4.2.4 ARCH-in-mean model. 83

4.2.5 Illustration. 84

4.3 GARCH model. 84

4.3.1 Forecasting. 88

4.3.2 Integrated GARCH model. 89

4.3.3 Estimation. 89

4.3.4 Testing for GARCH effects. 92

4.3.5 Software to estimate ARCH and GARCH models. 92

4.3.6 Illustration. 93

4.4 Asymmetric GARCH models. 94

4.4.1 EGARCH model. 94

4.4.2 TGARCH model. 95

4.4.3	GJR model	95
4.4.4	Cox-Box transform	95
4.4.5	News impact curve	96
4.4.6	Partially non-parametric estimation	96
4.4.7	Testing for asymmetric effects	97
4.4.8	Illustration	99
4.5	GARCH model with jumps	99
4.5.1	A model with time-varying jump intensity	101
4.5.2	An empirical illustration	105
4.6	Aggregation of GARCH processes	108
4.6.1	Temporal aggregation	109
4.6.2	Cross-sectional aggregation	113
4.6.3	Estimation of the weak GARCH process	114
4.7	Stochastic volatility	115
4.7.1	From GARCH models to stochastic volatility models	115
4.7.2	Estimation of the discrete time SV model	117
4.8	Realized volatility	118
4.8.1	The difficulty to disentangle jumps	119
4.8.2	Quadratic variation	123
4.8.3	Power variation	124
4.8.4	Bipower variation	126
4.8.5	Estimation over finite time intervals	128
4.8.6	Realized covariance	135
4.8.7	Further related results	141
	Modeling Higher Moments	143
5.1	The general problem	144
5.1.1	Higher moments of a GARCH process	145
5.1.2	Quasi Maximum Likelihood Estimation	148
5.1.3	The existence of distribution with given moments	151
5.2	Distributions with higher moments	152
5.2.1	Semi-parametric approach	153
5.2.2	Series expansion about the normal distribution	155
5.2.3	Skewed Student t distribution	159
5.2.4	Generating asymmetric distributions	166
5.2.5	Pearson IV distribution	169
5.2.6	Entropy distribution	172
5.3	Specification tests and inference	177
5.3.1	Moment specification tests	177
5.3.2	Adequacy tests based on density forecasts	179
5.3.3	Adequacy tests based on interval forecasts	180
5.4	Illustration	182
5.5	Modeling conditional higher moments	188
5.5.1	Tests for autoregressive conditional higher moments	189
5.5.2	Modeling higher moments directly	189

5.5.3	Modeling the parameters of the distribution	191
6	Modeling Correlation	195
6.1	Multivariate GARCH models	197
6.1.1	Vectorial and diagonal GARCH models	198
6.1.2	Dealing with large-dimensional systems	200
6.1.3	Modeling conditional correlation	206
6.1.4	Estimation issues	210
6.1.5	Specification tests	212
6.1.6	Test of constant conditional correlation matrix	214
6.1.7	Illustration	217
6.2	Modeling the multivariate distribution	223
6.2.1	Standard multivariate distributions	225
6.2.2	Skewed elliptical distribution	230
6.2.3	Skewed Student t distribution	233
6.2.4	Estimation	236
6.2.5	Adequacy tests	239
6.2.6	Illustration	240
6.3	Copula functions	240
6.3.1	Definitions and properties	241
6.3.2	Measures of concordance	242
6.3.3	Non-parametric copulas	244
6.3.4	Review of some copula families	245
6.3.5	Estimation	254
6.3.6	Adequacy tests	258
6.3.7	Modeling the conditional dependency parameter	259
6.3.8	Illustration	261
7	Extreme Value Theory	265
7.1	Univariate tail estimation	266
7.1.1	Distribution of extremes	266
7.1.2	Tail distribution	276
7.1.3	The case of weakly dependent data	291
7.1.4	Estimation of high quantiles	296
7.2	Multivariate dependence	300
7.2.1	Characterizing tail dependency	303
7.2.2	Estimation and statistical inference on x and x	307
7.2.3	Modeling dependency	308
7.2.4	An illustration	309
7.2.5	Further investigations	311

Part III Applications of Non-Gaussian Econometrics

8	Risk Management and VaR	315
8.1	Definitions and measures.	316
8.1.1	Definitions.	316
8.1.2	Models for portfolio returns.	320
8.2	Historical simulation.	321
8.3	Semi-parametric approaches.	322
8.3.1	Extreme Value Theory (EVT).	324
8.3.2	Quantile regression technique.	328
8.4	Parametric approaches.	330
8.4.1	RiskMetrics - J.P. Morgan.	331
8.4.2	The portfolio-level approach.	334
8.4.3	The asset-level approach.	337
8.5	Non-linear models.	341
8.5.1	The "delta-only" method.	341
8.5.2	The "delta-gamma" method.	341
8.6	Comparison of VaR models.	342
8.6.1	Evaluation of VaR models.	343
8.6.2	Comparison of methods.	343
8.6.3	10-day VaR and scaling.	344
8.6.4	Illustration.	345
9	Portfolio Allocation	349
9.1	Portfolio allocation under non-normality.	349
9.1.1	Direct maximization of expected utility.	350
9.1.2	An approximate solution based on moments.	353
9.2	Portfolio allocation under downside risk.	359
9.2.1	Definition.	360
9.2.2	Downside risk as an additional constraint.	360
9.2.3	Downside risk as an optimization criterion.	361

Part IV Option Pricing with Non-Gaussian Returns

10	Fundamentals of Option Pricing	365
10.1	Notations.	366
10.2	The no-arbitrage approach to option pricing.	369
10.2.1	Choice of a stock price process.	369
10.2.2	The fundamental partial differential equation.	371
10.2.3	Solving the fundamental PDE.	373
10.2.4	The Black-Scholes-Merton formula.	375
10.3	Martingale measure and BSM formula.	377
10.3.1	Self-financing strategies and portfolio construction.	377

- 10.3.2 Change of numeraire 378
- 10.3.3 Change of Brownian motion 378
- 10.3.4 Evolution of S_t under Q 379
- 10.3.5 The expected pay-off as a martingale 379
- 10.3.6 The trading strategies 380
- 10.3.7 Equivalent martingale measure 381

- 11 Non-structural Option Pricing 383**
 - 11.1 Difficulties with the standard BSM model 384
 - 11.2 Direct estimation of the risk-neutral density 385
 - 11.2.1 Expression for the RND 385
 - 11.2.2 Estimating the parameters of the RND 387
 - 11.3 Parametric methods 389
 - 11.3.1 Mixture of log-normal distributions 389
 - 11.3.2 Mixtures of hypergeometric functions 394
 - 11.3.3 Generalized beta distribution 395
 - 11.4 Semi-parametric methods 395
 - 11.4.1 Edgeworth expansions 395
 - 11.4.2 Hermite polynomials 399
 - 11.5 Non-parametric methods 402
 - 11.5.1 Spline methods 402
 - 11.5.2 Tree-based methods 406
 - 11.5.3 Maximum entropy principle 407
 - 11.5.4 Kernel regression 408
 - 11.6 Comparison of various methods 409
 - 11.7 Relationship with real probability 414
 - 11.7.1 The link between RNDs and objective densities 414
 - 11.7.2 Empirical findings 416

- 12 Structural Option Pricing 417**
 - 12.1 Stochastic volatility model 417
 - 12.1.1 The square root process 418
 - 12.1.2 Solving the PDE based on characteristic function 419
 - 12.1.3 A new partial differential equation 422
 - 12.2 Option pricing with stochastic volatility 425
 - 12.2.1 Hull and White (1987, 1988) 425
 - 12.2.2 Heston (1993) 426
 - 12.2.3 Characteristic function of the SV model 428
 - 12.2.4 Further insights 429
 - 12.3 Models with jumps 432
 - 12.3.1 Stochastic process with jumps 432
 - 12.3.2 Diffusion with double exponential jumps 434
 - 12.3.3 Combining stochastic volatility with jumps 436
 - 12.3.4 Jumpy affine models 440
 - 12.4 Models with even wilder jumps: Levy option pricing 441

12.4.1	Commonly used Levy processes	443
12.4.2	Choice of the time-changing process	444
12.4.3	Option pricing	445
12.4.4	Pricing options with risk-neutral characteristic function	446
12.4.5	Empirical results	447

Part V Appendices on Option Pricing Mathematics

13	Brownian Motion and Stochastic Calculus	451
13.1	Law of large numbers and the central limit theorem	451
13.2	Random walks	453
13.3	Construction of the Brownian motion	453
13.4	Properties of the Brownian motion	456
13.5	Stochastic integration	457
13.6	Stochastic differential equations	459
13.7	Ito's lemma	460
13.8	Multivariate extension of Ito's lemma	462
13.9	Transition probabilities and partial differential equations	463
13.10	Kolmogorov backward and forward equations	464
13.11	PDE associated with diffusions	466
13.12	Feynman-Kac formula	468
14	Martingale and Changing Measure	471
14.1	Martingales	471
14.2	Changing probability of a normal distribution	472
14.3	Radon-Nikodym derivative	473
14.4	Girsanov's theorem	474
14.5	Martingale representation theorem	475
15	Characteristic Functions and Fourier Transforms	477
15.1	Characteristic functions	477
15.1.1	Basic properties	478
15.1.2	Moments and the characteristic function	478
15.1.3	Convolution theorem	479
15.1.4	Uniqueness	480
15.1.5	Inversion theorem	480
15.2	Fourier transform and characteristic function	483
16	Jump Processes	487
16.1	Counting and marked point process	487
16.2	The Poisson process	489
16.2.1	Construction of the Poisson distribution	489
16.2.2	Properties of the Poisson distribution	491
16.2.3	Moments of pure Poisson process	492

16.2.4 Compound Poisson process	493
16.3 The exponential distribution	494
16.3.1 Definition and properties	494
16.3.2 Moments of the exponential variable	495
16.3.3 Hazard and survivor functions	496
16.4 Duration between Poisson jumps	497
16.5 Compensated Poisson processes	498
17 Levy Processes	501
17.1 Construction of the Levy process	501
17.2 Properties of Levy processes	505
References	507
Index	535